Circular Motion

Everything we've learned so far has a rotational equivalent. Next 2 chapters.

As we did for linear motion, first we develop the equations of motion.

Then we can discuss the forces that cause this motion.

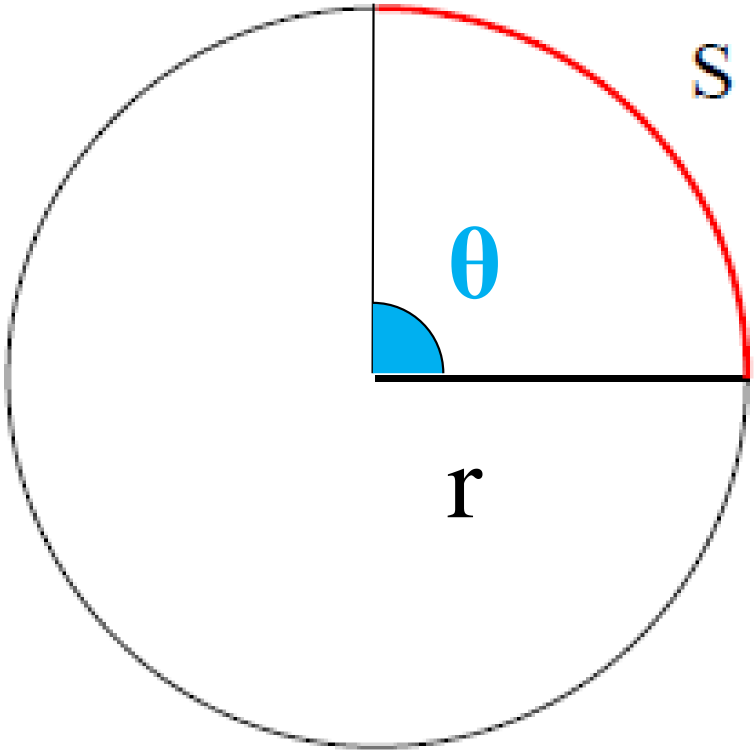
Today’s Objectives
Rotational Motion

After today, you should be able to:

- Convert Between Degrees/Radians/Revs
- Define angular velocity/acceleration
- Go between linear and angular quantities in order to solve problems

Practice: 7.1, 7.5, 7.7, 7.9, 7.11, 7.13, 7.15

Some Definitions



s = Arclength

r = radius of circle

θ = Angle

$$s = r\theta$$

What is *s* for one rotation (circumference of a circle)?

a) πr
b) $2\pi r$
c) πr^2
d) $2\pi r^2$

Q85

Radians

Circumference of a circle = $2\pi r$

Thus, $\theta = 2\pi$ for one revolution of a circle

What are the units of θ?

Notes:

$$\theta = \frac{s}{r}$$

θ measured in radians

2π radians = 360°

2π radians = 1 revolution

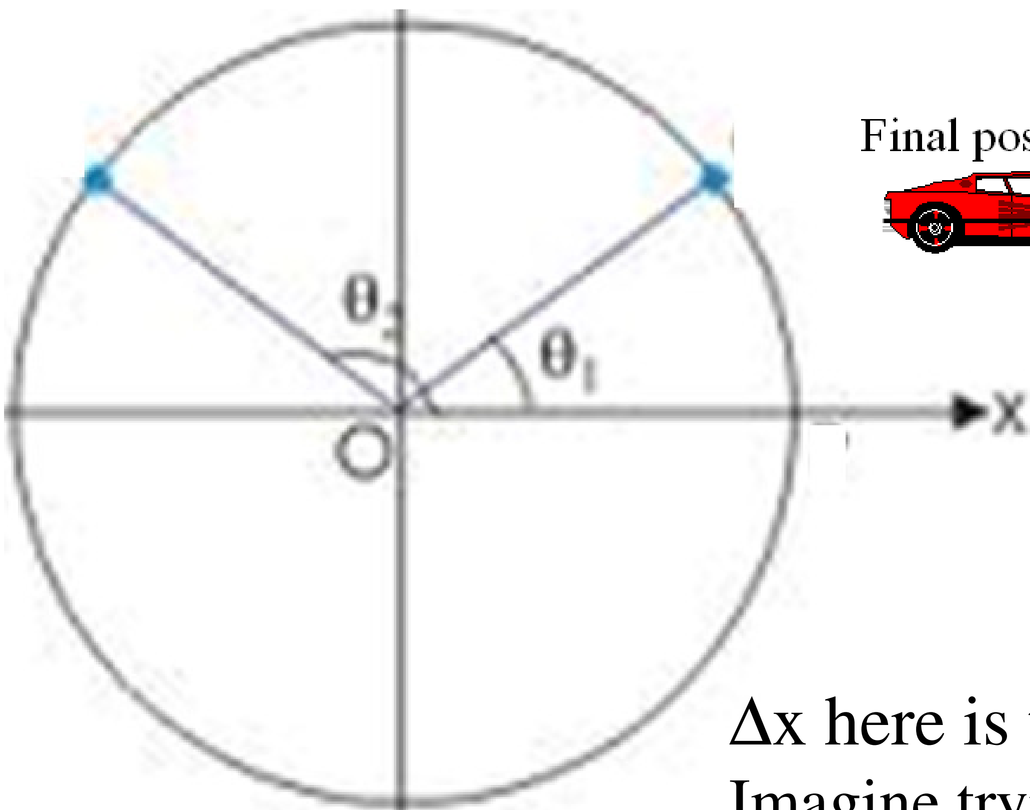
$$360°\left(\frac{2\pi\ radians}{360°}\right) = 2\pi\ radians$$

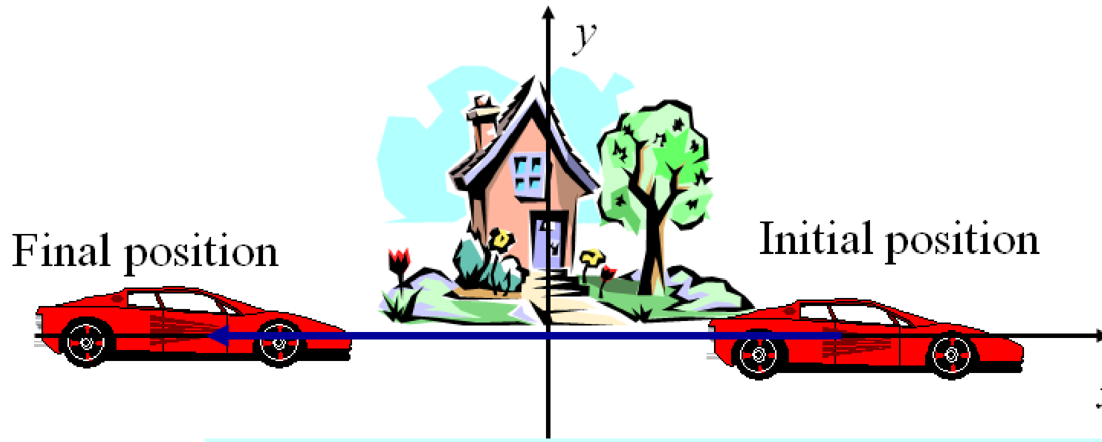
$\pi/3$ radians of a pizza = what is the angle?

Q86 A. 30 B. 60 C. 90 D. 120 E. 240

Angular Position vs Displacement

Just like we could define initial and final positions, to find linear displacement, we can do the same for angular displacement

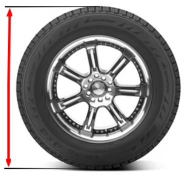




$$\Delta\theta = \theta_f - \theta_i = \frac{\Delta x}{r}$$

Δx here is the displacement along the arclength. Imagine trying to curve a ruler around a curved path.

The tires on a car have a diameter of 2.0 ft and are warranted for 60,000



miles. **Determine the angle** (in radians) through which one of these tires will rotate during the warranty period.

$$\Delta\theta = \frac{\Delta x}{r}$$

How many revolutions of the tire are equivalent to our answer?

How would this change if the tires were bigger?

Consider two people racing around a track.

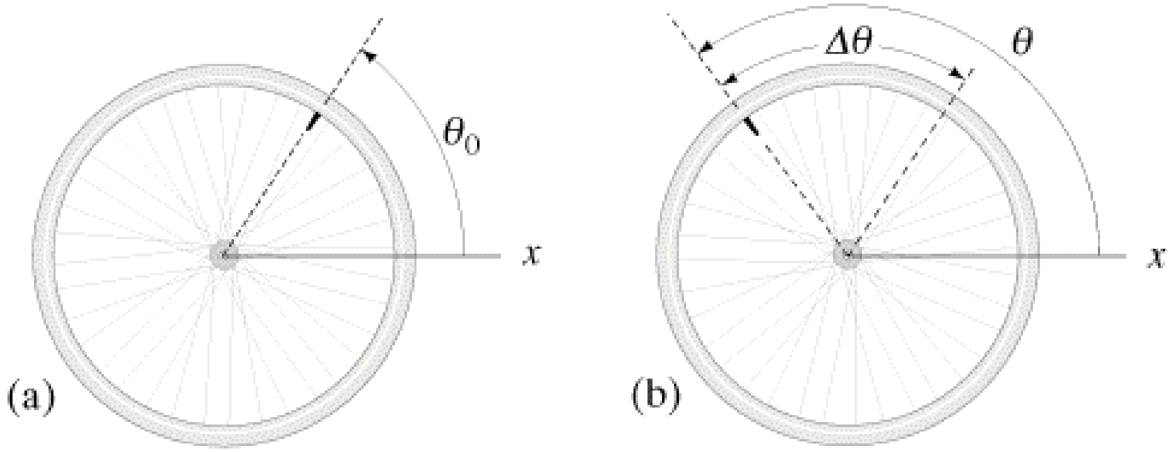
In a backyard race, which side (inner or outer) of the track would you rather be on?

$$\Delta x = r\Delta\theta$$

Q92

Rotational vs. Linear Motion

<u>Average</u> angular velocity:



$$\overline{\omega} = \frac{\Delta\theta}{\Delta t} \qquad \overline{v} = \frac{\Delta x}{\Delta t}$$

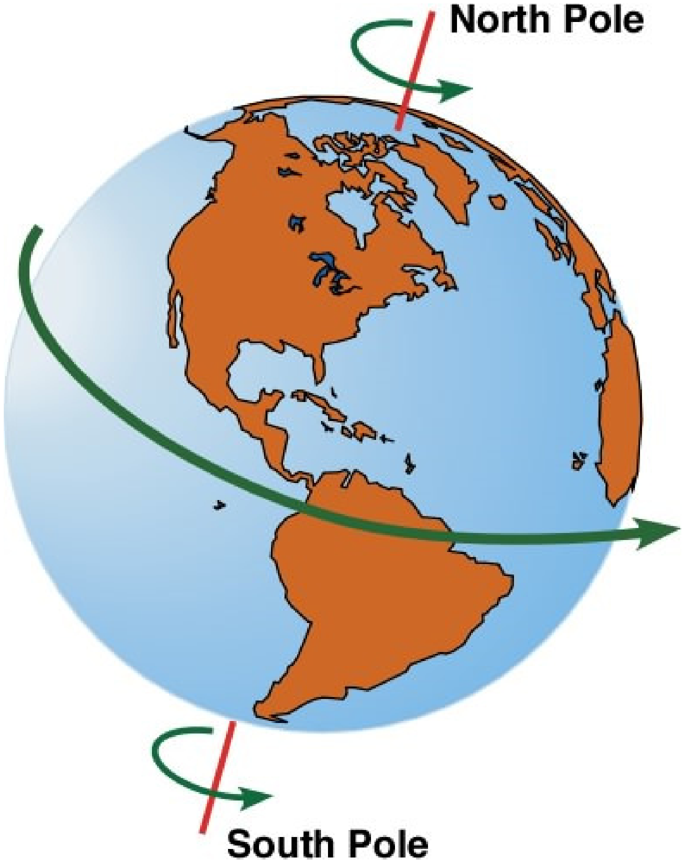
units: rad/s (m/s)

Average angular acceleration:

$$\overline{\alpha} = \frac{\omega - \omega_o}{\Delta t} = \frac{\Delta\omega}{\Delta t} \qquad \overline{a} = \frac{\Delta v}{\Delta t}$$

units: rad/s^2 (units: m/s^2)

Find the angular speed of Earth's rotation about its axis. (No #s!)



$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

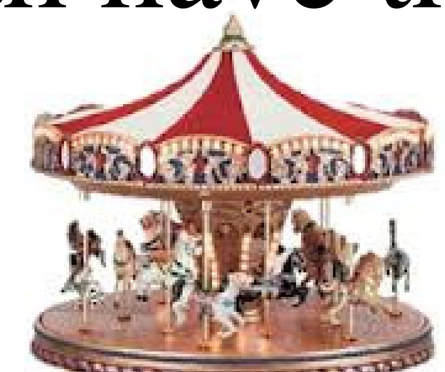
BIG BEN in London and a little alarm clock both keep perfect time. Which minute hand has the bigger angular velocity ω?

A) Big Ben

B) little alarm clock

C) Both have the same ω

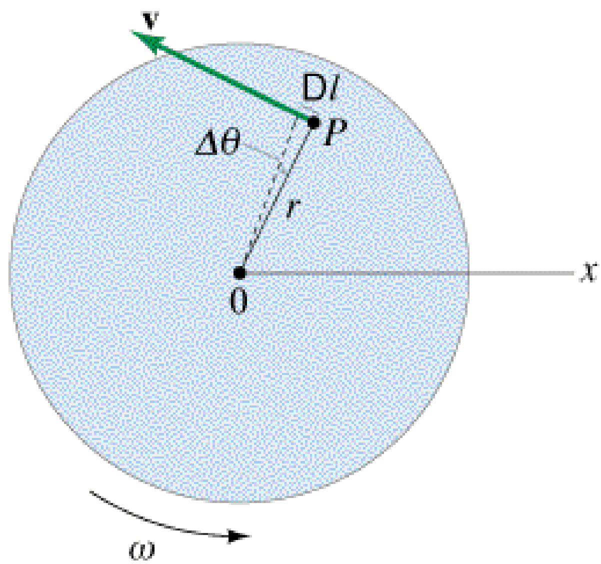
$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$



Q87

Relations Between Angular and Linear Quantities

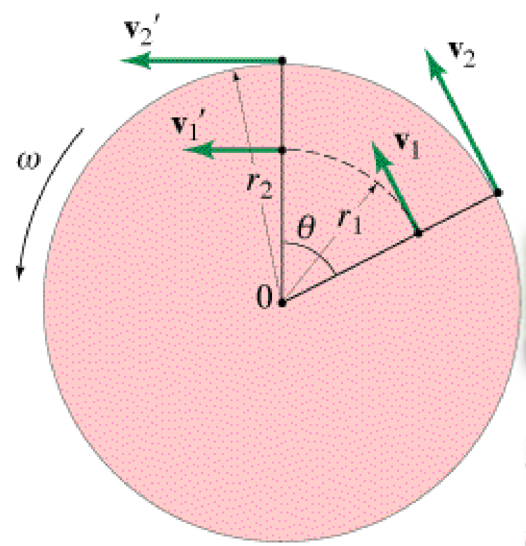
Linear and angular velocities



v is linear speed of point P

$$v = \frac{\Delta x}{\Delta t} = \frac{r\Delta\theta}{\Delta t}$$

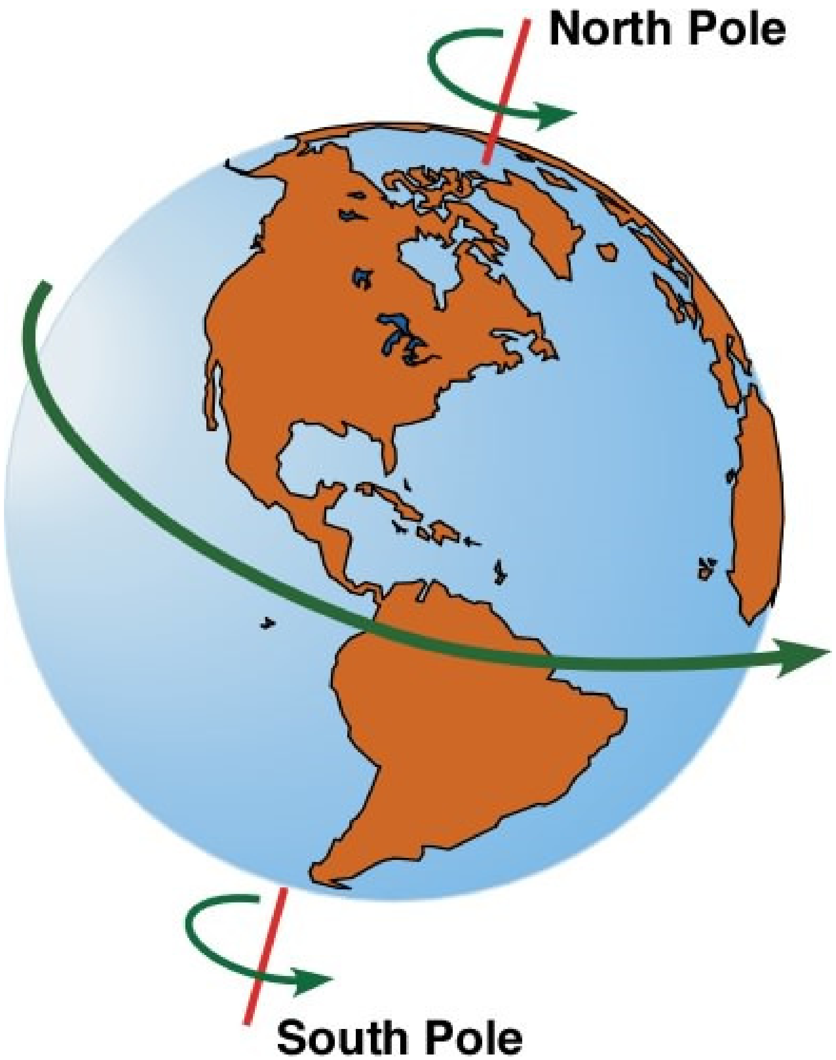
$$\Rightarrow \quad v = r\omega$$



On a merry go round, are you noticing the linear or angular velocity?

Note:

Since **<u>ω is the same for all points of a solid object undergoing rotational motion</u>**, v is proportional to distance (r) of point from center of rotation



What is the linear speed of an object at rest on Earth's surface at the equator with respect to Earth's center? (Earth's radius is ~ 6400 km.)

$$v = r\omega$$

Relating Ch.2 to Rotational Motion

Same process as Ch. 2: List what you know in variable form, then match with a formula

Linear	Rotational	
Δx	$\Delta\theta$	$\Delta x = r\Delta\theta$
$\bar{v} = \dfrac{\Delta x}{\Delta t}$	$\bar{\omega} = \dfrac{\Delta\theta}{\Delta t}$	$v = r\omega$
$\bar{a} = \dfrac{\Delta v}{\Delta t}$	$\bar{\alpha} = \dfrac{\Delta\omega}{\Delta t}$	$a = r\alpha$

Note the line means average!

For constant a:

$$v = v_o + at$$

$$\Delta x = v_o t + \tfrac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

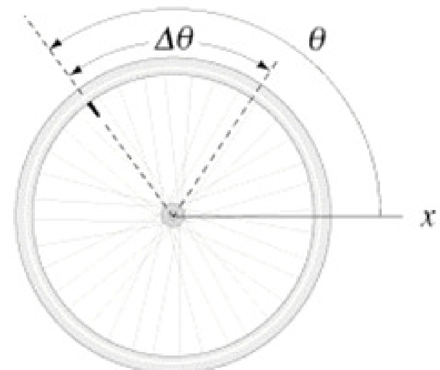
If the radius does not change

For constant α:

$$\omega = \omega_o + \alpha t$$

$$\Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$$



A student sees the following question on an exam:

A flywheel with mass 120 kg, and radius 0.6 m, starting at rest, has an angular acceleration of 0.1 rad/s^2. How many revolutions has the wheel undergone after 10 s?

Which formula should the student use to answer the question?

A. $\omega = \omega_o + \alpha t$

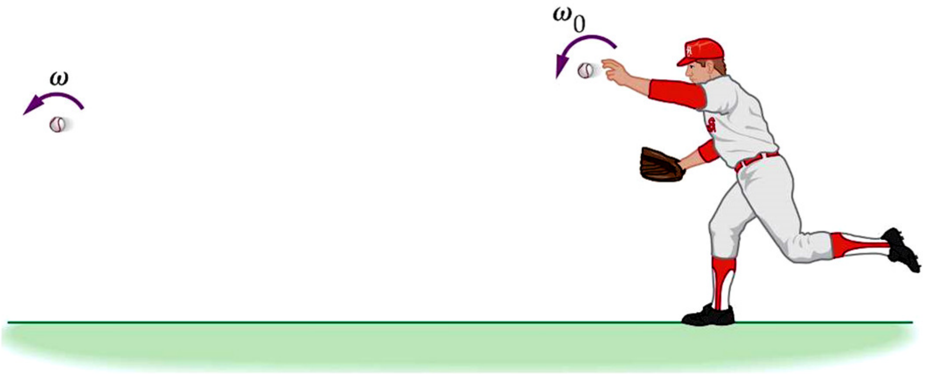
B. $\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$

C. $\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$

Q88

Example: Thrown for a Curve

To throw a curve ball, a pitcher gives the ball an initial angular speed of 36.0 rad/s. When the catcher gloves the ball 0.595 s later, its angular speed has decreased (due to air resistance) to 34.2 rad/s.



Part b: You could also use average angular velocity formula, but have to use average!
As in Ch. 2, many people forget to use average.

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

(a) What is the ball's angular acceleration, assuming it to be constant?

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{(34.2 \text{ rad/s}) - (36.0 \text{ rad/s})}{(0.595 \text{ s})} = -3.03 \text{ rad/s}^2$$

$$\omega = \omega_o + \alpha t$$

$$\Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

(b) How many revolutions does the ball make before being caught?

$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$= (36.0 \text{ rad/s})(0.595 \text{ s}) + \tfrac{1}{2}(-3.03$$

$$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$$

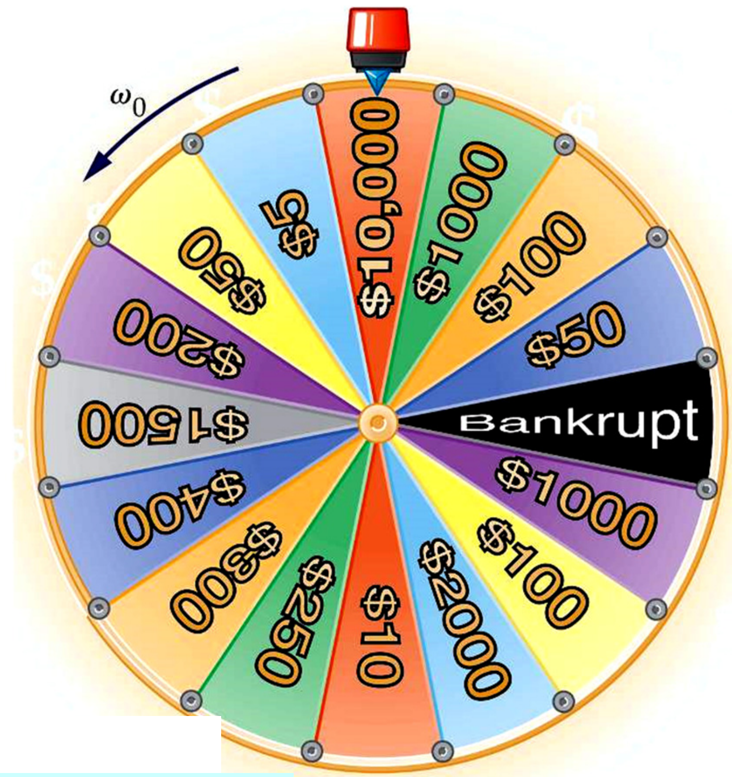
$$= 20.9 \text{ rad} = 3.33 \text{ rev}$$

Wheel of Misfortune

Amy is on the Wheel of Fortune and has to spin the wheel. She gives the wheel an initial angular speed of 3.40 rad/s. It then rotates through 1.25 revolutions and comes to rest on BANKRUPT.

(a) Find the wheel's angular acceleration, assuming it is constant.

(b) How long does it take for the wheel to stop?



$$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta}$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (3.40\text{ rad/s})}{(-0.736\text{ rad/s}^2)} = 4.62\text{ s}$$

My kids’ tricycle

Predict

I put stickers on the bottom of the front and back wheels of different sizes. As I roll the bike (without slipping), the stickers complete a circle (360 degrees) at:

a) The same time
b) Different times
c) Depends on the speed of the bike

Q90

$$\omega = \omega_o + \alpha t$$

$$\Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$$

A DVD is initially at rest so that the line *PQ* on the disc's surface is along the +*x*-axis. The disc begins to turn with a constant $\alpha = 5.0$ rad/s^2.

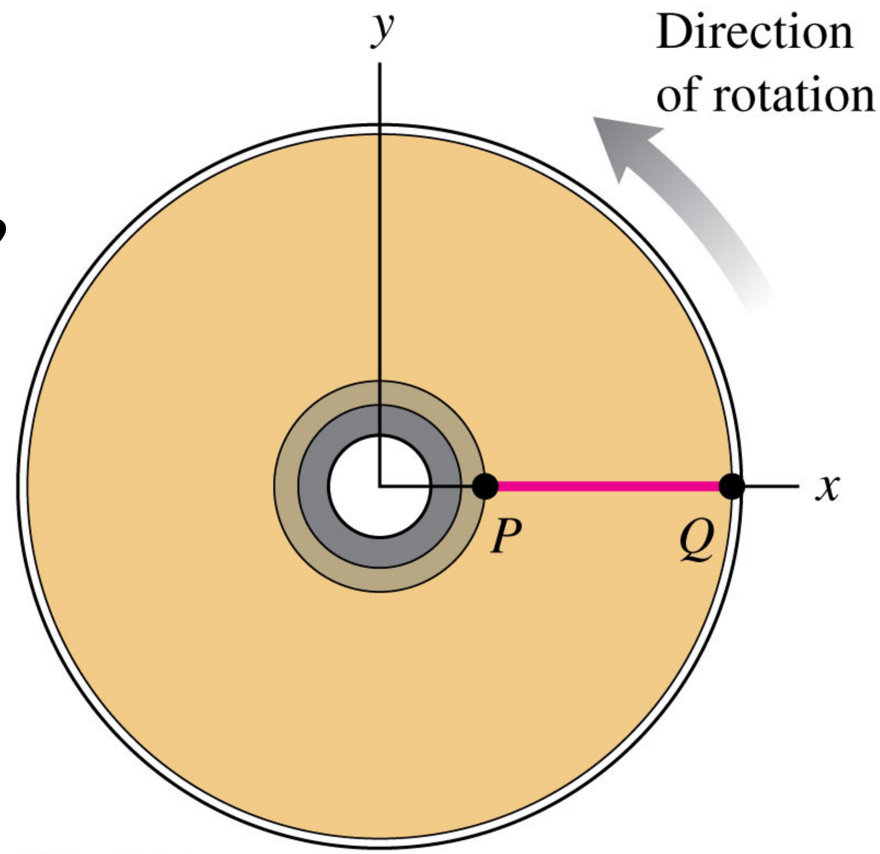
At $t = 0.40$ s, what is the angle between the line *PQ* and the +*x*-axis?

A. 0.40 rad

B. 0.80 rad

C. 1.0 rad

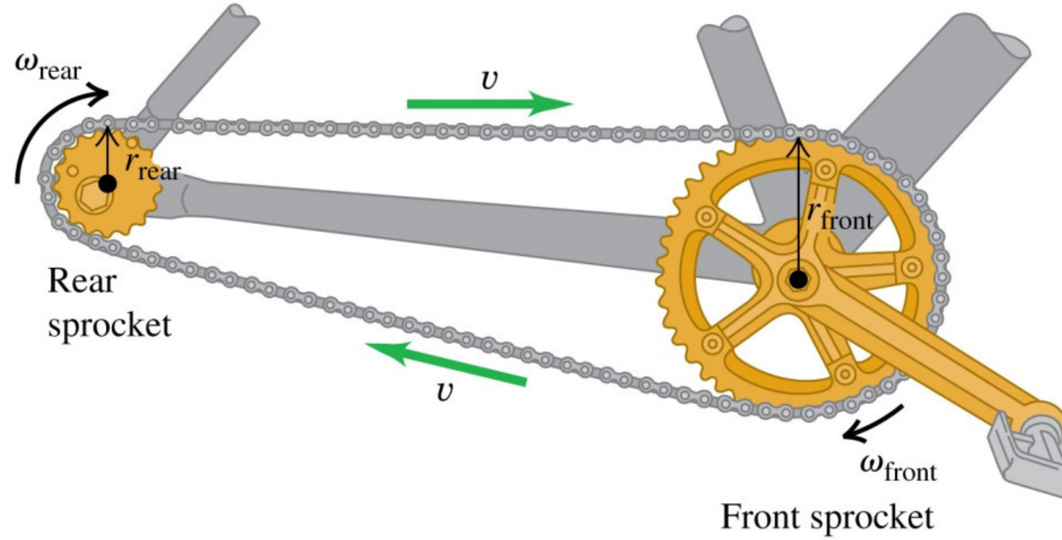
D. 2.0 rad



$$\theta = \frac{x}{r} \qquad \overline{\omega} = \frac{\Delta\theta}{\Delta t} \qquad \overline{\alpha} = \frac{\Delta\omega}{\Delta t}$$

Q89

Compared to a gear tooth on the rear sprocket (on the left, of small radius) of a bicycle, a gear tooth on the *front* sprocket (on the right, of large radius) has



A. a faster linear speed and a faster angular speed.

B. the same linear speed and a faster angular speed.

C. a slower linear speed and the same angular speed.

D. the same linear speed and a slower angular speed.

E. none of the above

Q91

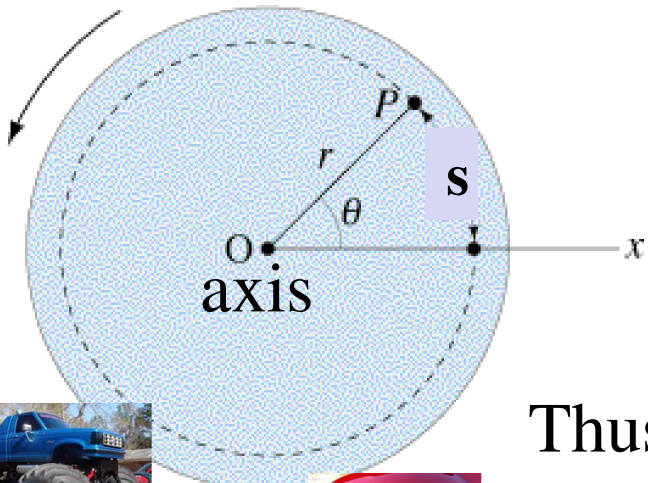
Hint: Figuring out what is constant

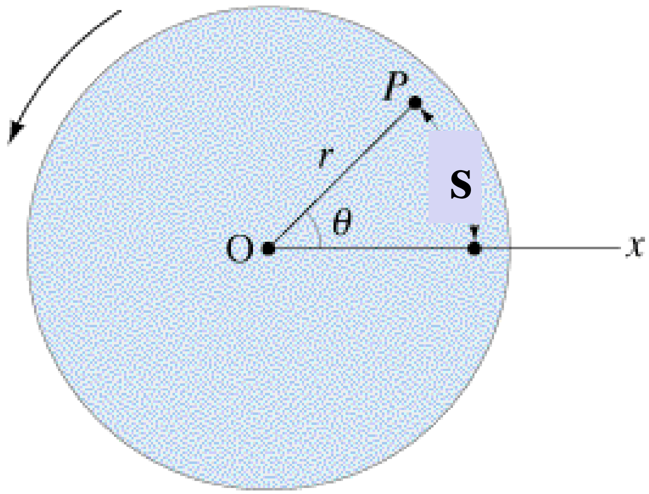
Angular Quantities

– Every point in a solid rotating about an axis moves in a circle

– Describe motion of that point by:

Angular position:





$$\Delta\theta = \frac{\Delta x}{r}$$

Thus:

$\Delta x = r\,\Delta\theta$

– A point closer to the center (smaller r) will have a smaller arclength for the same angle

– Or, **if Δx is constant**, a small wheel makes more turns for same distance ($\Delta\theta = \Delta x/r$)

Clicker Answers

85=B, 86=B, 87=C, 88=B, 89=A, 90=B, 91=D, 92=B